

Discrete Fourier Transform in Nanostructures using Scattering

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In this paper we show that the discrete Fourier transform can be performed by scattering a coherent particle or laser beam off a two-dimensional potential that has the shape of rings or peaks. After encoding the initial vector into the two-dimensional potential, the Fourier-transformed vector can be read out by detectors surrounding the potential. The wavelength of the laser beam determines the necessary accuracy of the 2D potential, which makes our method very fault-tolerant.

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I. INTRODUCTION

The fast developing fields of quantum computing and quantum information processing have attracted much interest recently¹⁻⁶. Most of the papers in the field of quantum computing, such as Shor's and Grover's about factoring numbers⁷ and searching a database⁸, use many-particle physics (i.e. qubits) for their implementation. However, it is also possible to implement quantum information processes in the so-called unary representation of dimension N , which consists of a statistical ensemble of non-interacting N -level systems that can be described by a single-particle Hamiltonian. So only interference, but no entanglement is needed for the implementation of quantum algorithms in the unary representation. It was already shown that the Grover algorithm⁹ can be performed in a single shot using the unary representation. While in Ref. 10 the quantum database search has been done experimentally with atoms in a beam that serve as non-interacting N -level systems, Refs. 11 and 12 show theoretically that the large spin of molecular magnets and nuclear spins in semiconductors can be used as non-interacting N -level systems, respectively. Here we propose a method to perform the classical discrete Fourier transform (DFT) in the time ensemble of quantum or classical scattering events, where the scattering potential provides the classical N -level system for the read-in of information and the N detectors provide the classical N -level system for the read-out of the information. In the case of quantum scattering a single-particle Hamiltonian describes the information processing. In contrast to Shor's algorithm our approach requires an exponential overhead of the hardware. However, once our device is built the DFT can be computed very efficiently, i.e. in a single shot. In our method we propose to scatter a coherent matter or laser beam off a two-dimensional potential consisting of N concentric rings or N peaks, the amplitudes of which represent the elements of a vector in an N -dimensional vector space where the information is encoded. The Fourier transformation of this vector can then be read out immediately by detectors surrounding

the 2D potential. Our method is closely related to the general field of Fourier optics¹³⁻¹⁵, where a diffraction pattern is Fourier transformed into a desired image pattern by means of laser scattering. We go a step further in the sense that we describe how to use a 2D scattering potential to perform the DFT efficiently. The concentric rings and peaks can be produced by charge or spin distributions. The calculation time τ_c is roughly given by the diameter of the detector ring (e.g. 1 mm) divided by the light velocity $c = 3 \times 10^8$ m/s, i.e. $\tau_c \sim 10^{-11}$ s. Using for example a laser with a wavelength of 500 nm and a spot radius of 100 μm enables the Fourier transformation of a function with 100 sampling points. The larger the spot size and the smaller the wavelength of the laser, the more sampling points can be achieved. The advantage of this device would be that it can be used already for example as DFT coprocessor to speed up present computers operating at room temperature. Rough estimates show a possible speedup of 10^6 or more (see below).

In Sec. II we give an overview on the DFT and the Fast Fourier Transform (FFT). Sec. III and Sec. IV describe the quantum-mechanical coherent matter scattering and the classical laser beam scattering off a specially designed 2D potential that is used for the implementation of the DFT.

II. DISCRETE FOURIER TRANSFORMATION

The discrete Fourier transformation (DFT) has a wide range of applications in the field of digital signal processing, such as spectral analysis and filtering. The DFT enables the possibility to extract the period of a function, which can be used to factor numbers. Thus finding efficient ways to compute the DFT is the key to the efficient factorization of numbers. The calculation time of a DFT can be greatly reduced by the classical algorithm called Fast Fourier Transform (FFT), which gets rid of the redundancies found in the DFT. In a similar way Shor's quantum algorithm reduces the calculation time of the discrete quantum Fourier transformation (DQFT)

by means of an efficient quantum circuit⁷ that uses qubits as information carriers. In our proposal the superposition principle of quantum or classical wavefunctions is used to perform an N -dimensional DFT with N shots or even a single shot in one scattering device. The FFT provides then the possibility to link several scattering devices in parallel. This means that the speedup factor N can be multiplied by the number of the parallel scattering devices.

First it is instructive to give the definition of a DFT. When we calculate the DFT of a function $f(z)$ that maps complex numbers on complex numbers, we can first discretize the function $f(z)$ in M total sampling points z_1, z_2, \dots, z_M , leading to M values $f_1 = f(z_1), f_2 = f(z_2), \dots, f_M = f(z_M)$. Then the DFT of f is given by

$$F_k = \sum_{j=0}^{M-1} e^{2\pi i j k / M} f_j. \quad (1)$$

The matrix elements of a DFT consist of the irreducible representation of the cyclic group produced by a rotation $R_{2\pi/M}$ about the angle $2\pi/M$, where M is the dimension of the vector space, which is equal to the number of sampling points. For example a DFT of dimension $M = 4$ is represented by

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{\frac{1}{4}2\pi i} & e^{\frac{1}{2}2\pi i} & e^{\frac{3}{4}2\pi i} \\ 1 & e^{\frac{1}{2}2\pi i} & e^{2\pi i} & e^{\frac{1}{2}2\pi i} \\ 1 & e^{\frac{3}{4}2\pi i} & e^{\frac{1}{2}2\pi i} & e^{\frac{1}{4}2\pi i} \end{bmatrix} \quad (2)$$

It will turn out that the exponential overhead of the hardware imposes a physical limit to the number of sampling points N that can be processed by our DFT device. However, the *Danielson-Lanczos Lemma*¹⁶, which is widely used for the FFT algorithm, states that the DFT for M sampling points can be separated into the sum over the DFT for the $M/2$ even sampling points and the DFT for the $M/2$ odd sampling points, i.e.

$$F_k = \sum_{j=0}^{M/2-1} e^{2\pi i k (2j) / M} f_{2j} + \sum_{j=0}^{M/2-1} e^{2\pi i k (2j+1) / M} f_{2j+1}. \quad (3)$$

Usually, the FFT uses this *Danielson-Lanczos Lemma* recursively down to Fourier transforms of length 2^0 . Therefore it is essential that $M = 2^m$, where m is a positive integer. As we want to use 2D scattering potentials with real amplitudes, we can separate the real and imaginary part of the DFT:

$$F_k = \sum_{j=0}^{M-1} e^{2\pi i j k / M} g_j + i \sum_{j=0}^{M-1} e^{2\pi i j k / M} h_j, \quad (4)$$

where $f(z_j) = g(z_j) + ih(z_j)$ with real functions $g(z_j)$ and $h(z_j)$.

Since our device can perform the DFT efficiently for N sampling points, we can stop the recursion at $N = 2^n$ sampling points, where n is a positive integer that is smaller than m . Using our device as coprocessor of an existing computer, the remaining work of the main processor consists in combining the M/N partial Fourier transforms, which takes of order $\log_2(M/N)M/N = (m-n)M/N$ operations. Thus, compared with initially necessary $M \log_2(M)$ operations a speedup of at least a factor N is achieved. In this estimate of the speedup, the necessary time to calculate the DFT for the N sampling points is assumed to be of the order of 10 ps, i.e. negligible. Alternatively, one could say that an existing computer equipped with our device is capable of factoring numbers that are N times larger than the largest nowadays factorizable number. If M/N DFT devices are linked in parallel, a speedup of factor M is achieved.

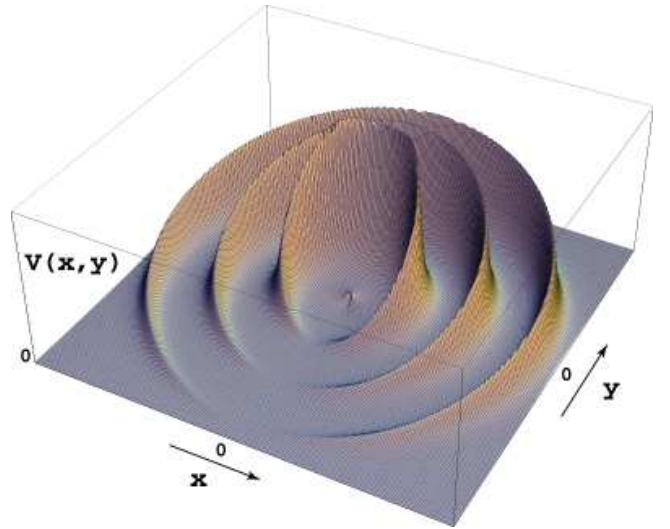


FIG. 1. 2D scattering potential $V(\mathbf{x}') = (1 + \sin \alpha) \sum_{j=1}^N a_j \delta(r' - r_j) / \sqrt{r'}$, which performs the DFT in N shots in the case of elastic scattering and in a single shot in the case of inelastic scattering.

III. COHERENT MATTER SCATTERING

Now we describe the various possible implementations of our DFT device, in which a coherent matter or laser beam is scattered off a 2D potential. This potential consists of radial δ -functions or δ -peaks on a straight line, the amplitudes of which both sample the points of the real or imaginary part of our function $f(z)$ and represent the amplitudes of our basis vectors. Let $\mathbf{v} = \sum_{j=1}^N a_j \mathbf{w}_j$ be the input vector, where \mathbf{w}_j are the basis vectors with amplitudes $a_j = g(z_j)$ or $a_j = h(z_j)$. Then our 2D potential is of the form $V(\mathbf{x}) \propto \sum_{j=1}^N a_j \delta(\mathbf{x} - \mathbf{x}_j)$. The read-out is done by detectors surrounding this potential. While the coherent matter beam needs a high precision of the potential, the laser beam provides a high fault-tolerance of

the potential since faults that are smaller than the wavelength of the laser beam remain undetected. We give the derivation for both the coherent matter and the laser beam scattering. We first start with the coherent matter beam. We use the nomenclature of Ref. 17. First we calculate the 2D scattering amplitude, from which we can infer the precise shape of the 2D potential required for the implementation of the DFT. For this we have to start from the well-known Lippman-Schwinger equation¹⁷

$$\langle \mathbf{x} | \psi^{(\pm)} \rangle = \langle \mathbf{x} | \phi \rangle + \frac{2m}{\hbar^2} \int d^2x' G_{\pm}(\mathbf{x}, \mathbf{x}') \langle \mathbf{x}' | V | \psi^{(\pm)} \rangle, \quad (5)$$

where $|\phi\rangle$ is the incoming wave, V is the 2D scattering potential, and

$$G_{\pm}(\mathbf{x}, \mathbf{x}') = \frac{\hbar^2}{2m} \langle \mathbf{x} | \frac{1}{E - H_0 \pm i\epsilon} | \mathbf{x}' \rangle \quad (6)$$

is the Green's function, which also solves the Helmholtz equation $(\nabla^2 + k^2)G_{\pm}(\mathbf{x}, \mathbf{x}') = \delta^{(2)}(\mathbf{x} - \mathbf{x}')$. For the evaluation of the Green's function we transform first to the momentum representation, i.e.

$$G_{\pm}(\mathbf{x}, \mathbf{x}') = \frac{\hbar^2}{2m} \int \frac{d^2p'}{(2\pi\hbar)^2} \frac{e^{\frac{i}{\hbar}\mathbf{p}' \cdot (\mathbf{x} - \mathbf{x}')}}{E - \frac{p'^2}{2m} \pm i\epsilon}, \quad (7)$$

where we have used the 2D overlap $\langle \mathbf{x} | \mathbf{p} \rangle = e^{i\mathbf{p} \cdot \mathbf{x} / \hbar} / 2\pi\hbar$. Inserting $E = \hbar^2 k^2 / 2m$, $\mathbf{p}' = \hbar \mathbf{Q}$, and going to polar coordinates (Q, ϕ) , we get

$$\begin{aligned} G_{\pm}(\mathbf{x}, \mathbf{x}') &= \frac{1}{(2\pi)^2} \int_0^{\infty} Q dQ \int_0^{2\pi} d\phi \frac{e^{iQ|\mathbf{x} - \mathbf{x}'| \cos \phi}}{k^2 - Q^2 \pm i\epsilon} \\ &\approx \frac{1}{(2\pi)^2} \int_0^{\infty} Q dQ \frac{2^{3/2} \sqrt{\pi}}{\sqrt{Q|\mathbf{x} - \mathbf{x}'|}} \\ &\quad \times \frac{\cos(Q|\mathbf{x} - \mathbf{x}'| - \frac{\pi}{4})}{k^2 - Q^2 \pm i\epsilon} \\ &= -\frac{1}{4\sqrt{\pi}} \frac{e^{\pm i(k|\mathbf{x} - \mathbf{x}'| - \frac{\pi}{4})}}{\sqrt{2k|\mathbf{x} - \mathbf{x}'|}}. \end{aligned} \quad (8)$$

For the approximation we have used the integral representation of the Bessel function $J_0(Q|\mathbf{x} - \mathbf{x}'|) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{iQ|\mathbf{x} - \mathbf{x}'| \cos \phi} \approx \sqrt{\frac{2}{\pi Q|\mathbf{x} - \mathbf{x}'|}} \cos(Q|\mathbf{x} - \mathbf{x}'| - \frac{\pi}{4})$, which holds in the asymptotic regime $Q|\mathbf{x} - \mathbf{x}'| \gg 1$, i.e. the local 2D potential $\langle \mathbf{x}' | V | \mathbf{x}'' \rangle = V(\mathbf{x}') \delta^{(2)}(\mathbf{x}' - \mathbf{x}'')$ and the detectors at \mathbf{x} are far apart from each other. Then we obtain with Eq. (5)

$$\langle \mathbf{x} | \psi^{(+)} \rangle = \frac{1}{2\pi} \left[e^{i\mathbf{k} \cdot \mathbf{x}} + \frac{e^{i(kr - \frac{\pi}{4})}}{\sqrt{kr}} f(\mathbf{k}', \mathbf{k}) \right], \quad (9)$$

where $r := |\mathbf{x}|$, $r' := |\mathbf{x}'|$, $\mathbf{k}' := k \frac{\mathbf{x}}{|\mathbf{x}|}$, and

$$f(\mathbf{k}', \mathbf{k}) = -\frac{\sqrt{2\pi}}{4} \frac{2m}{\hbar^2} \int d^2x' e^{-i\mathbf{k}' \cdot \mathbf{x}'} V(\mathbf{x}') \langle \mathbf{x}' | \psi^{(+)} \rangle \quad (10)$$

is the 2D scattering amplitude. The amplitude of the 2D spherical wave decays as $1/\sqrt{r}$. This is in agreement with the topological argument that the circumference of a circle is $2\pi r$ and that the probability has to be conserved on the circle. In first-order Born approximation, where $|f(\mathbf{k}', \mathbf{k})| \ll \sqrt{kr}$, we can insert $\langle \mathbf{x}' | \psi^{(+)} \rangle = \frac{e^{i\mathbf{k} \cdot \mathbf{x}'}}{2\pi}$ into Eq. (10), and we define $\mathbf{q} := \mathbf{k}' - \mathbf{k}$ as the scattering wavevector. Eq. (10) becomes then

$$f^{(1)}(\mathbf{k}', \mathbf{k}) = -\frac{1}{4\sqrt{2\pi}} \frac{2m}{\hbar^2} \int d^2x' e^{iqx' \cos \varphi} V(\mathbf{x}'), \quad (11)$$

where φ is the angle between \mathbf{q} and \mathbf{x}' .

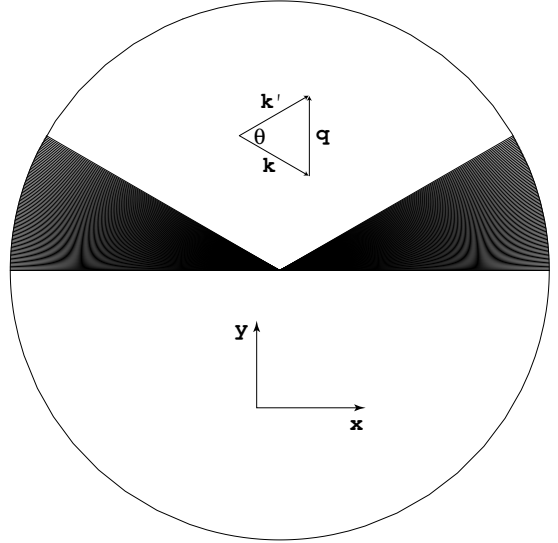


FIG. 2. If the scattering is elastic, the DFT can be performed with N shots. The 2D potential shown in Fig. 1 is located in the center of the ring. The length of the lines before and after the scattering are proportional to the length of the wavevectors \mathbf{k} and \mathbf{k}' , respectively.

For the implementation of the DFT we start first with a 2D potential of the form $V(\mathbf{x}') = V(r')(1 + \cos \varphi)$. If the scattering is elastic, we choose $\varphi = \pi/2 - \theta/2$. If the material allows inelastic scattering, we choose $\varphi = \pi/2 - \theta$. It will turn out that for this potential \mathbf{q} has to point always into the same direction. For example \mathbf{q} can be always perpendicular to the x -axis. Then we can define α as the angle between the x -axis and \mathbf{x}' , which turns our potential into $V(\mathbf{x}') = V(r')(1 + \sin \alpha)$. This 2D potential is shown in Fig. 1. Inserting this potential into Eq. (11) leads to

$$f^{(1)}(\mathbf{k}', \mathbf{k}) = -\frac{\sqrt{2\pi}}{4} \frac{2m}{\hbar^2} \int r' dr' [J_0(qr') + iJ_1(qr')] V(r'), \quad (12)$$

which in the asymptotic limit $r' \gg 1/q$ becomes

$$f^{(1)}(\mathbf{k}', \mathbf{k}) = -\frac{1}{2\sqrt{q}} \frac{2m}{\hbar^2} \int \sqrt{r'} dr' e^{i(qr' - \frac{\pi}{4})} V(r'). \quad (13)$$

By choosing $V(r') = \sum_{j=1}^N a_j \delta(r' - r_j) / \sqrt{r'}$ we can encode the information into the N amplitudes a_j and perform a DFT

$$f^{(1)}(\mathbf{k}', \mathbf{k}) = -\frac{1}{2\sqrt{q_\nu}} \frac{2m}{\hbar^2} e^{-i\frac{\pi}{4}} \sum_{j=1}^N a_j e^{iq_\nu r_j} \quad (14)$$

In order for it to be a *discrete* Fourier transformation (DFT), we have to discretize the variables. Let us make a first attempt: $r_j = j\frac{l}{N}$, $j = 1, 2, \dots, N$, and $q_\nu = \nu\frac{2\pi}{l}$, $\nu = 1, 2, \dots, N$, with ν referring to the ν th detector. The smallest optically resolvable distance between the radii r_j is $\lambda/2$. So it is safe to choose $l = N\lambda$, where $\lambda = 2\pi/k$ is the deBroglie wavelength of the coherent beam. Then we obtain $\frac{l}{N} = 2\sin\frac{\theta}{2}$ for the elastic scattering and $\frac{l}{N} = \tan\theta$ for the inelastic scattering (see below). The radii of the rings are multiples of the wavelength λ , i.e. $r_j = j\lambda$. There is still the requirement $r_j \gg 1/q_\nu$ that has to be satisfied. Therefore we have to choose $j = N+1, N+2, \dots, 2N$, which still produces a DFT, since the DFT is 2π -periodic.

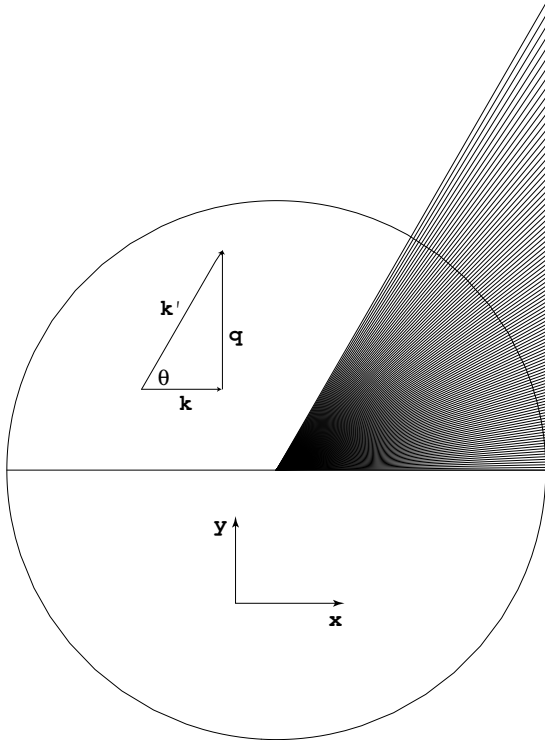


FIG. 3. The 2D scattering potential shown in Fig. 1 is located at the center of the ring. The length of the lines before and after the scattering are proportional to the length of the wavevectors \mathbf{k} and \mathbf{k}' , respectively. The DFT can be performed with a single shot if the scattering off the 2D scattering potential is inelastic.

Our 2D potential $V(\mathbf{x}')$ has the $1 + \cos\varphi$ dependence, so that we obtain J_0 and J_1 Besselfunctions in Eq. (12). Since we do not want to adjust the orientation of $V(\mathbf{x}')$

according to the scattering vector \mathbf{q} , we have to keep the direction of \mathbf{q} constant in space. This requirement can be satisfied in two ways, which can be identified by elastic or inelastic scattering. In the case of elastic scattering \mathbf{k} and \mathbf{k}' have the same length. So $q := |\mathbf{k} - \mathbf{k}'| = 2k \sin\frac{\theta}{2}$, where θ is the angle between \mathbf{k} and \mathbf{k}' . For every incoming beam of angle $\vartheta_{\text{in}} = \theta/2$ the resulting vector element after the DFT can be read out in the detector at the outgoing angle $\vartheta_{\text{out}} = \vartheta_{\text{in}} = \theta/2$. The number of incoming and outgoing angles is exactly N . Thus the DFT can be performed with N shots, which is shown in Fig. 2. In the case of inelastic scattering, \mathbf{k} and \mathbf{k}' do not have the same length anymore. It is then possible to perform the DFT in a single shot. For this $q := |\mathbf{k} - \mathbf{k}'| = k \tan\theta$. Since $k' = n_r k$ and $k = k' \cos\theta$, we obtain an angle-dependent refractive index $n_r = 1/\cos\theta$, which is probably difficult to realize experimentally. At least in the case of a laser beam (see below) it may be possible to use photonic crystals that mimic $n_r = 1/\cos\theta$. The inelastic scattering is shown in Fig. 3.

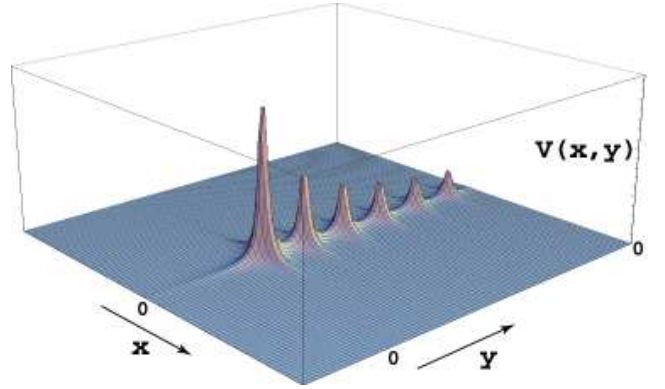


FIG. 4. 2D scattering potential $V(\mathbf{x}') = \sum_{j=1}^N a_j \delta(r' - r_j) \delta(\alpha - \alpha_j) / r'$, which performs the DFT in a single shot for elastic scattering.

Instead of $V(\mathbf{x}') = (1 + \sin\alpha) \sum_{j=1}^N a_j \delta(r' - r_j) / \sqrt{r'}$ consisting of concentric rings, we can alternatively use peaks of the form $V(\mathbf{x}') = \sum_{j=1}^N a_j \delta(r' - r_j) \delta(\alpha - \alpha_j) / r'$, which is shown in Fig. 4. After putting the peaks on a straight line ($\alpha_1 = \alpha_2 = \dots = \alpha_N = \pi/2$), we obtain from Eq. (11)

$$f^{(1)}(\mathbf{k}', \mathbf{k}) = -\frac{1}{4\sqrt{2\pi}} \frac{2m}{\hbar^2} \sum_{j=1}^N a_j e^{iq_\nu r_j \cos\varphi_\nu}, \quad (15)$$

where $\alpha_\nu = \varphi_\nu + \pi/2$ is the angle between the scattering vector \mathbf{q}_ν and the x-axis. With this potential we can discretize the variables similarly as before: $r_j = j\frac{l}{N}$, $j = 1, 2, \dots, N$, and $q_\nu \cos\varphi_\nu = \nu\frac{2\pi}{l}$, $\nu = 1, 2, \dots, N$. The advantage of this method is that the peaks can be much easier produced experimentally than the rings. Also, we can start from the peak with $j = 1$, because we do not require $r_j \gg 1/q_\nu$ anymore. There is also no scaling factor $1/\sqrt{q_\nu}$ needed for the detectors (see Eq. (14)) and

the scattering can be done elastically with a single shot (see Fig. 5) because the scattering vectors \mathbf{q}_ν do not need to point into the same direction. However, the potential made of peaks decreases as $1/r'$, and not as $1/\sqrt{r'}$ as for the rings.

For the experimental realization of the 2D scattering off a 2D potential one could use a 2D electron gas that is scattered off an electrostatic potential. Like in the case of the laser beam scattering (see next section), it would be also possible to use 3D scattering off a 2D potential, such as the scattering of spin-polarized neutrons off a spatial electronic spin distribution.

IV. LASER BEAM SCATTERING

Since coherent matter waves have a very short de-Broglie wavelength, the resolution of the 2D potential must be very high ($< 1 \text{ \AA}$), which is beyond today's technology. In order to overcome this difficulty, one can use 3D scattering of a laser beam off a 2D potential, where the wavelength of the laser can be chosen according to the accuracy of the 2D potential. For this we have to start from Maxwell's equations. Following Ref. 18 we need to solve the Helmholtz equation

$$\begin{aligned} (\nabla^2 + k^2)\mathbf{D} &= -\nabla \times \nabla \times (\mathbf{D} - \epsilon_0\mathbf{E}) \\ &\quad - \frac{i\epsilon_0\omega}{c}\nabla \times (\mathbf{B} - \mu_0\mathbf{H}), \end{aligned} \quad (16)$$

where \mathbf{D} and \mathbf{B} are the electrostatic and magnetic fields within the sample, \mathbf{E} and \mathbf{H} are the respective fields of the laser beam, ϵ_0 is the dielectric constant, μ_0 is the permeability, and $k = \sqrt{\epsilon_0\mu_0\omega}/c$ is the wavevector of our laser beam. The result reads¹⁸

$$\mathbf{D} = \mathbf{D}^{(0)} + \mathbf{A}_s e^{ikr}/r, \quad (17)$$

where the scattering amplitude in Born approximation is given by

$$\begin{aligned} \frac{\boldsymbol{\epsilon}^* \cdot \mathbf{A}_s^{(1)}}{D_0} &= \frac{k^2}{4\pi} \int d^3x' e^{i\mathbf{q}\cdot\mathbf{x}'} \left\{ \boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0 \frac{\delta\epsilon(\mathbf{x}')}{\epsilon_0} \right. \\ &\quad \left. + (\mathbf{n} \times \boldsymbol{\epsilon}^*) \cdot (\mathbf{n}_0 \times \boldsymbol{\epsilon}_0) \frac{\delta\mu(\mathbf{x}')}{\mu_0} \right\}, \end{aligned} \quad (18)$$

where $\delta\epsilon(\mathbf{x}') \ll \epsilon_0$ and $\delta\mu(\mathbf{x}') \ll \mu_0$. Here \mathbf{n}_0 and $\boldsymbol{\epsilon}_0$ are the direction and the polarization of the incoming beam, \mathbf{n} and $\boldsymbol{\epsilon}$ are the direction and the polarization of the outgoing beam, respectively. Let us restrict ourselves to variations in the dielectric constant $\delta\epsilon(x')$, i.e. $\delta\mu(x') = 0$. In analogy to the 2D potential for the coherent matter beam, we can use a dielectric variation of the ring form $\delta\epsilon(\mathbf{x}') = (1 + \sin\alpha) \sum_{j=1}^N a_j \delta(r' - r_j)/\sqrt{r'}$ or of the peak form $\delta\epsilon(\mathbf{x}') = \sum_{j=1}^N a_j \delta(r' - r_j) \delta(\alpha - \alpha_j)/r'$. Performing the integration in cylindrical coordinates, $\int d^3x' = \int r' dr' d\varphi dz$, for the peak form leads to

$$\frac{\boldsymbol{\epsilon}^* \cdot \mathbf{A}_s^{(1)}}{D_0} = \frac{k^2 d}{4\pi\epsilon_0} \boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0 \sum_{j=1}^N a_j e^{iq_\nu r_j \cos\varphi_\nu}, \quad (19)$$

where d is the thickness of the sample. So 3D scattering off a 2D potential provides another means to perform the DFT.

For the experimental implementation the spatial dielectric variation $\delta\epsilon(\mathbf{x}')$ can be produced for example electrostatically by means of the Kerr effect. As another example one could use a crystal with spin-orbit interaction where an imbalance of spin-up and spin-down electrons leads to different dielectric constants for left and right circularly polarized light²². In this case a circularly polarized laser beam is scattered off a spatial dielectric variation $\delta\epsilon(\mathbf{x}')$ that is created by a spatial electronic spin distribution. It is also worth to mention that the laser scattering can be done at room temperature.

Now we estimate the amount of sampling points ($=N$) that can be used with today's technology. The state-of-the-art lasers of e.g. $\lambda = 500 \text{ nm}$ wavelength produce an intensity of $J=1 \text{ GW/cm}^2$ at the sample. This implies a photon number density of $n = \frac{J}{ch\omega} \sim 10^{10}$ photons per $(100 \mu\text{m})^3$. If only 10^{-2} of the laser beam is scattered, about 10^4 photons remain at the detectors surrounding the sample at $r = 1 \text{ mm}$. Within a radius of $100 \mu\text{m} = 2N\lambda$ we can insert $N = 100$ rings or peaks, i.e. use a basis of $N = 100$ sampling points.

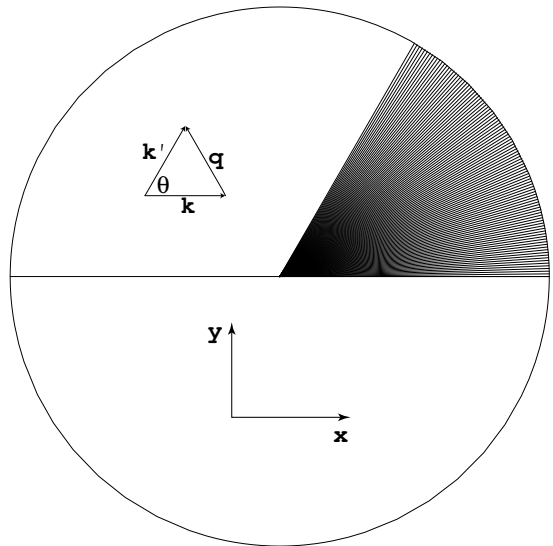


FIG. 5. The 2D scattering potential shown in Fig. 4 is located in the center of the ring. The length of the lines before and after the scattering are proportional to the length of the wavevectors \mathbf{k} and \mathbf{k}' , respectively. The DFT can be performed with a single shot by using elastic scattering off the 2D scattering potential.

Until now we have taken a 2D variation of the dielectric constant $\delta\epsilon(\mathbf{x}') = \sum_{j=1}^N a_j \delta(r' - r_j) \delta(\alpha - \alpha_j)/r'$. For experiments it is more realistic to assume a 2D varia-

tion of the dielectric constant $\delta\epsilon(\mathbf{x}') = \sum_{j=1}^N a_j \delta^{(w)}(r' - r_j) \delta^{(w)}(\alpha - \alpha_j) / wr'$ made of delta functions of width w , such as $\delta^{(w)}(r' - r_j) = w/\pi[(r' - r_j)^2 + w^2]$. Then Eq. (19) becomes approximately

$$\begin{aligned} \frac{\boldsymbol{\epsilon}^* \cdot \mathbf{A}_s^{(1)}}{D_0} &= \frac{k^2 d}{4\pi\epsilon_0} \boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0 \sum_{j=1}^N a_j \int dr'' \frac{w e^{iq_\nu(r''+r_j) \cos \varphi_\nu}}{\pi(r''^2 + w^2)} \\ &= \frac{k^2 d e^{-q_\nu | \cos \varphi_\nu | w}}{4\pi\epsilon_0} \boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0 \sum_{j=1}^N a_j e^{iq_\nu r_j \cos \varphi_\nu}, \quad (20) \end{aligned}$$

where we have made the substitution $r'' = r' - r_j$. As long as $w \ll 1/2k = \lambda/4\pi \leq 1/q_\nu |\cos \varphi_\nu|$, this exponential damping factor is negligible. Using a laser of wavelength $\lambda = 500$ nm implies that $w \ll 50$ nm. It is also possible to scale up the proposed setup to longer wavelengths without changing the results. One could say that the required accuracy of the 2D potential is given by the wavelength of the laser. Therefore our scheme is very flexible and very fault-tolerant.

Since $\frac{\nu}{N} = 2 \sin \frac{\theta}{2}$ for elastic scattering, the detectors are to be arranged within a scattering angle of $\pi/3$. The smallest angle between two detectors is $\theta_{\min} = \pi/3N = \pi/300$, which leads to a minimum distance of $r\theta_{\min} = 10$ μm between the detectors.

V. CONCLUSION

In conclusion, we have shown that by means of scattering a coherent matter or laser beam off a 2D potential it is possible to perform the DFT on N sampling points. So the scattering is done quantum-mechanically or classically. The 2D potential can be made of concentric rings or peaks on a straight line. The scattering off the peaks should be much easier to implement experimentally since it can be done elastically with a single shot. Note that the DFT is performed classically in both cases since we use a classical 2D potential to encode classical information. This provides the great advantage that our device can already be used as DFT coprocessor for existing classical computers, without having to worry about the decoherence of quantum states, like in the case of quantum computing or spintronics devices^{19–23}. In addition, M/N devices can be linked together to perform the DFT in parallel on M sampling points, thereby making use of the FFT. It should be possible with today's technology to build a DFT device with the capability to Fourier transform an $N = 1024$ -dimensional vector. If an existing classical computer was equipped with $M/N = 1024$ such DFT devices that are linked together, a speedup of a factor $M = 1024N \sim 10^6$ could be achieved.

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